Computer Graphics

Chapter_4

2D Transformations

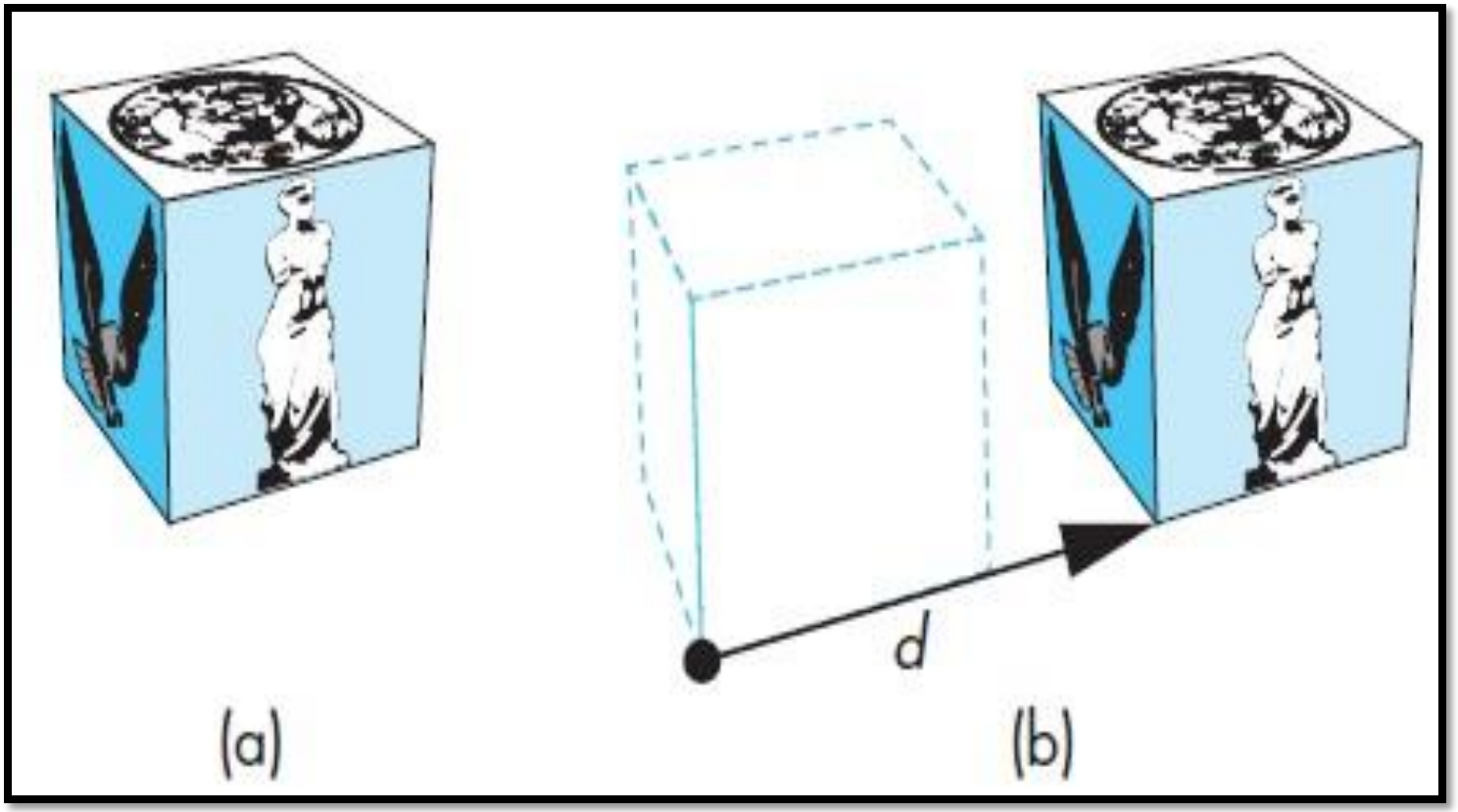
Lecture #1

Translation

- Translation is an operation that displaces points by a fixed distance in a given direction.
- To specify a translation, we need only to specify a displacement vector ***d***, because the transformed points are given by:

$$P' = P + d$$

- P refer to all points on the object in its **old position**.
- P' refer to all points on the object in its **new position**.



(a) Object in its original position.

(b) Translated Object.

- We translate a two-dimensional point by adding translation distances , $\boldsymbol{t_x}$ and $\boldsymbol{t_y}$, to the original coordinate position $(\boldsymbol{x}, \boldsymbol{y})$ to move the point to a new position $(\boldsymbol{x'}, \boldsymbol{y'})$.

$$x' = \boldsymbol{x} + \boldsymbol{t_x}$$

$$y' = \boldsymbol{y} + \boldsymbol{t_y}$$

- The translation distance pair $\left(\boldsymbol{t_x}, \boldsymbol{t_y}\right)$ is called a translation vector or shift vector.

- We can express the translation equations as a single matrix equation by using column vectors to represent coordinate positions and the translation vector.
- Translation is a ***rigid-body transformation*** that moves objects without deformation.
- That is, every point on the object is translated by the same amount.

- A straight Line segment is translated by applying the transformation equation to each of the line endpoints and redrawing the line between the new endpoint positions.
- Polygons are translated by adding the translation vector to the coordinate position of each vertex and regenerating the polygon using the new set of vertex coordinates and the current attribute settings.

- Similar methods are used to translate curved objects.
- To change the position of a circle or ellipse, we translate the center coordinates and redraw the figure in the new location.
- We translate other **curves** (for example, splines) by displacing the coordinate positions defining the objects, then we reconstruct the curve paths using the translated coordinate points.

- Expressing positions in homogeneous coordinates allows us to represent all geometric transformation equations as matrix multiplications.
- Coordinates are represented with three-element column vectors, and transformation operations are written as 3×3 matrices.

- For Translation. we have:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- The translation matrix can be written in the abbreviated form:

$$\boldsymbol{P' = T(t_x, t_y).\ P}$$

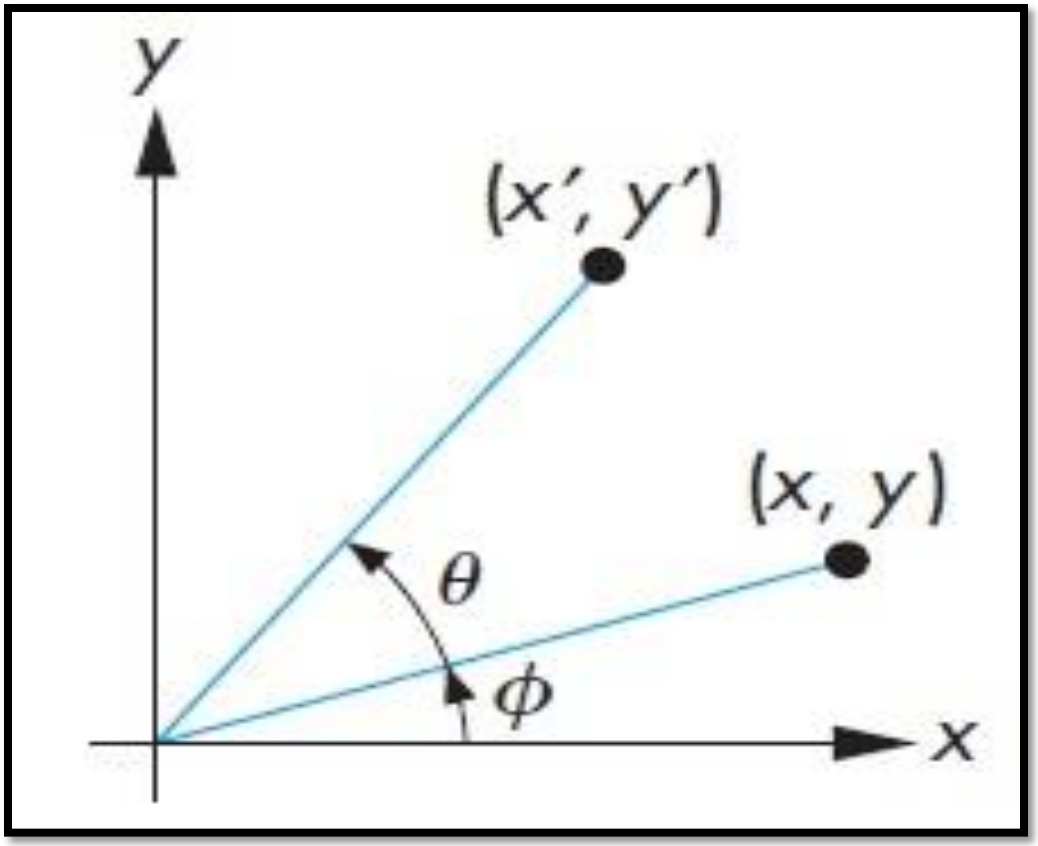
with $\boldsymbol{T(t_x, t_y)}$ as the 3 by 3 translation matrix.

- The inverse of the translation matrix is obtained by replacing the translation parameters $\boldsymbol{t_x}$ and $\boldsymbol{t_y}$ with their negatives: $\boldsymbol{-t_x}$ and $\boldsymbol{-t_y}$.

Rotation

- A two-dimensional rotation is applied to an object by repositioning it along a circular path in the **xy** plane.
- To generate a rotation, we specify a rotation angle θ and the position $(\boldsymbol{x_r}, \boldsymbol{y_r})$ of the rotation point (or pivot point) about which the object is to be rotated.
- Positive values for the rotation angle define counterclockwise rotations about the pivot point.

- Negative values rotate objects in the clockwise direction.
- This transformation can also be described as a rotation about a rotation axis that is perpendicular to the **xy** plane and passes through the pivot point.



Two-dimensional rotation.

- Having specified a particular point—the origin—we are in a particular frame.
- A two-dimensional point at (x, y) in this frame is rotated about the origin by an angle **θ** to the position (x', y'). We can obtain the standard equations describing this rotation by representing (x, y) and (x', y') in polar form:

$$x = \rho \cos\varphi,$$
$$y = \rho \sin\varphi,$$

$$x' = \rho \cos(\theta + \varphi),$$
$$y' = \rho \sin(\theta + \varphi),$$

- Expanding these terms using the trigonometric identities for the sine and cosine of the sum of two angles, we find:

$$x' = \rho \left[\cos(\theta)\cos(\varphi) - \sin(\theta)\sin(\varphi)\right],$$
$$y' = \rho \left[\cos(\theta)\sin(\varphi) + \sin(\theta)\cos(\varphi)\right],$$
$$x' = x \cos\theta - y \sin\theta$$
$$y' = x \sin\theta + y \cos\theta$$

- These equations can be written in matrix form as:
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$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

- Expressing positions in homogeneous coordinates allows us to represent all geometric transformation equations as matrix multiplications.

- Coordinates are represented with three-element column vectors, and transformation operations are written as 3×3 matrices.
- For Rotation:

we have: $\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & \mathbf{0} \\ \sin\theta & \cos\theta & \mathbf{0} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

- The rotation matrix can be written in the abbreviated form:

$$\boldsymbol{P' = R(\theta).\ P}$$

with $\boldsymbol{R(\theta)}$ as the 3 by 3 Rotation matrix.

- The inverse of the Rotation matrix is obtained by replacing the rotation angle $\boldsymbol{\theta}$ with its negative $\boldsymbol{-\theta}$.

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